Techniques For Obfuscation:

Relaxations and New Notions

by

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Layout of Talk

1. Relaxed definitions of obfuscation
   - Special purpose obfuscators
   - Predicate obfuscators
2. Some positive results
   - Point function obfuscation (Random oracles)
   - Point function obfuscation (w/o ROs)
   - Some predicate obfuscators
3. Some more notions of obfuscation

Preliminaries

- For any two programs ($\boldsymbol{P_1}$, $\boldsymbol{P_2}$), we say $\boldsymbol{P_1 \sim P_2}$ if both programs have identical functionality
  - Also extends to "approximate" functionality (For all but a negligible fractions of inputs, both programs have identical outputs)
- We say any Boolean predicate π of the programs ($\boldsymbol{P_1}$, $\boldsymbol{P_2}$) is a semantic predicate if

$$\boldsymbol{P_1 \sim P_2} \text{ implies } \pi(\boldsymbol{P_1}) = \pi(\boldsymbol{P_2})$$

Obfuscation [Barak et al.]

Obfuscator is an algorithm $\boldsymbol{O}$ s.t. for all programs $\boldsymbol{P}$:

- *Functionality:* ***O(P) ~ P***
- *Poly Slowdown:*
  - ***Size(O(P)) < Poly (Size(P))***
  - ***Time(O(P)) < Poly(Time(P))***
- *Virtual Black-Box:* For all semantic predicates π, and for all algorithms ***A***, there exists a simulator ***S*** such that:

$$\Pr[\boldsymbol{A}(\boldsymbol{O}(\boldsymbol{P}))= \pi(\boldsymbol{P})] - \Pr[\ \boldsymbol{S}^{\boldsymbol{P}} = \pi(\boldsymbol{P})] \approx 0$$

- Such an obfuscator ***O*** cannot exist! [Barak01]

  By constructing a "cannibalistic" program that says:

  "Feed me somebody that behaves like me, and I'll leak my secret!"

What can we hope to achieve?

We relax some of the requirements of [Barak01]

Obfuscator is an algorithm $\boldsymbol{O}$ s.t. for certain programs $\boldsymbol{P}$:

- *Functionality:* $\boldsymbol{O(P) \sim P}$
- *Poly Slowdown:*
  - $\boldsymbol{Size(O(P)) < Poly\ (Size(P))}$
  - $\boldsymbol{Time(O(P)) < Poly(Time(P))}$
- *Virtual Black-Box:* For certain semantic predicates π, and for all algorithms $\boldsymbol{A}$, there exists a simulator $\boldsymbol{S}$ such that:

$$\Pr[\boldsymbol{A}(\boldsymbol{O}(\boldsymbol{P})) = \pi(\boldsymbol{P})] - \Pr[\boldsymbol{S}^{\boldsymbol{P}} = \pi(\boldsymbol{P})] \approx 0$$

- "Special purpose obfuscator" and/or "Predicate obfuscator"

More Relaxations….

- Allow Random Oracles (RO)
- RO is a type of “black-box” with true randomness
  - Infeasible to predict output on some input without making an explicit query to the RO
  - Infeasible to find collisions (2 inputs give same output)
- In the RO model, all participants (obfuscator, attacker and obfuscated program) have access to the random oracle.
- Lynn et al. use RO to securely obfuscate certain predicates of point functions. [Lynn04]

Point Function Obfuscation [Lynn04]

- A point function outputs 1 at only one input and 0 otherwise
- Password checking programs: (Password is “hello world!”)

```
VERIFY_PASSWORD (Input X){
    If (X==“hello world!”) Then output 1;
    Else output 0;
}
```

Let Random_Oracle(“hello world!”) = 813841341

```
VERIFY_PASSWORD_OBF (Input X){
    If (Random_Oracle(X) == 813841341) Then output 1;
    Else output 0;
}
```

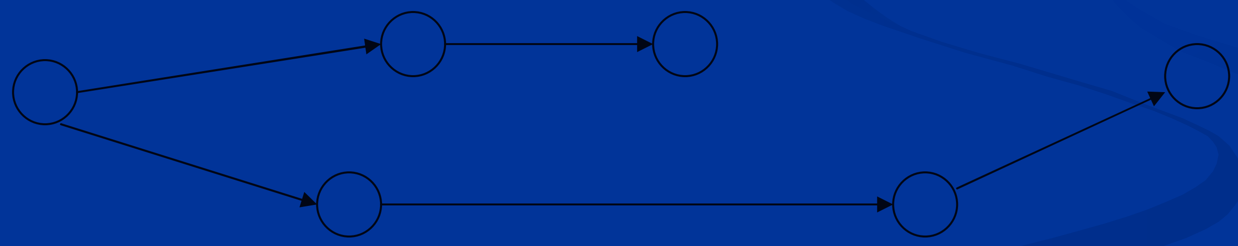
- Let π_i(VERIFY_PASSWORD) denote i^{th} bit of password.
- Provably secure obfuscation of predicate π_i for all i.
- Obfuscation preserves approximate functionality!

Point Functions with output [Lynn 04]

- Instead of 1, the function on some input *a* outputs some value *b* > 1
  - Obfuscation: Use two random oracles. Generate random *r* and store { *r* , Random_Oracle1 (*a* , *r*) , Random_Oracle2 (*a*, *r*) XOR *b* }
- Multi-point functions with output: Many point functions with output:

$$F_{A,\,B}(x) = B_i \;\; if \;\; x = A_i$$

  - Obfuscation: Repeat above for each input/output pair (with different *r*)
- Multi-point functions for "Access control" (via directed graphs)
  - Edge *i* has "password" A_i needed to access secret B_i at head node
  - Can only access some node if we can prove a path from start node.



- The above method to obfuscate a multi-point function with output is secure assuming single point function obfuscation is secure [Lynn04]
  - Key idea is a "composition of obfuscations" Lemma

Point Functions [Wee05]

- [Wee05] uses the basic idea of [Lynn04] for obfuscating point functions with random oracles.
- Gives an instantiation of random oracle under assumption that a certain type of one-way permutation exists.
- These types of one-way permutations are believed to exist (eg. RSA)
- One of the few constructions where a random oracle can be instantiated by a real function.
- *Caveat:* Technique of [Lynn04] to convert point function obfuscation to multi-point function obfuscation fails!
  - "composition of obfuscations" Lemma does not work for [Wee05]

Predicate Obfuscation (some more)

(Learnable v/s non-learnable)

Two fundamental types of programs

- Learnable: (can re-create source code just from few I/O queries)

```
Program_1 (input X){
        /* Ignore input */
        Output 0;
}
```

- Not learnable: (cannot re-create source code from few I/O queries)

```
Program_2 (input X){
        If (X == 1668801023012013) Then
            Output 1;
        Else   Output 0;
}
```

- Predicate π (P) : To decide if program P is learnable or not.
- In other words, given Program_i for unknown i ⟵ {1, 2}, to decide:
  - Does there exist X such that Program_i(X) = 1 ?

Predicate Obfuscation …
(learnable v/s non-learnable)

- From previous slide, Program_2 (non-learnable) contains some "hidden" functionality inside, while Program_1 (learnable) does not.
- *Applications:* (perhaps) watermarking [Varnovsky03] (watermarked program contains some hidden functionality)
- *Goal:* Want to hide the predicate π that indicates if the program can ever output 1 or not.
- [Varnovsky03] give a method for hiding which program (from previous slide) is given.
- Their construction is based on any one-way function and information theoretic.
- We will give (for simplicity) a construction using a number-theoretic primitive. We assume that for a composite n, with unknown factorization,
    1. Computing square roots mod n is as hard as factoring n
    2. For any $1 < x < n-1$, such that Jacobi_Symbol $(x, n)=1$, deciding if x is a quadratic residue mod n is hard.

learnable v/s non-learnable…

```
Program_1 (input X){
        /* Ignore input */
       Output 0;
}
```

```
Program_2 (input X){
    If (X == 1668801023012013) Then
                        Output 1;
    Else Output 0;
}
```

- Let k be the bit-length of X such that Program_2(X) = 1
- Generate $n = pq$ for large primes p, q (assume $|k| = |n|$). Let $y = X^2 \bmod n$ and let w be a quadratic non-residue mod n such that Jacobi_symbol(w, n)=1.

```
Obf_Program_1 (input X){
      const w ;
      If (X^2 mod n == w ) Then
                  Output 1;
      Else Output 0;
}
```

```
Obf_Program_2 (input X){
      const y ;
      If (X^2 mod n == y ) Then
                  Output 1;
      Else Output 0;
}
```

- Since w is a quadratic non-residue, Obf_Program_1 will never output 1.
- Without factors of n we cannot decide if w (or y) is a quadratic residue or not.
- Hence provably secure obfuscation of the predicate π.

More notions of obfuscation

- **Indistinguishability obfuscation [Barak01]**
  - If $P_1 \sim P_2$, then the obfuscations $O(P_1)$ **and** $O(P_2)$ are indistinguishable
  - Not clear how much information is "hidden"!
- **Best-possible obfuscation [Goldwasser07]**
  - The obfuscation $O(P)$ leaks as little information as possible, and is therefore the "best possible"
  - Informally, any other program $P' \sim P$ with $|P'| \leq |O(P)|$ leaks more information than $O(P)$
  - Formal definitions given for circuits (but we will skip this)
  - Mostly negative results! ☹

Obfuscating Re-Encryption

Re-encryption for asymmetric ciphers

- Given a ciphertext encrypted under Alice's encryption key, transform it into a ciphertext under Bob's encryption key (without knowing Alice's decryption key). Thus, some sort of obfuscation is required.
- [Hohenberger07] gave an obfuscation for re-encryption using bilinear maps.
- Uses a slightly different notion of obfuscation
  - "Average case secure obfuscation" – Indistinguishability of the output of an adversary with access to obfuscated code and that of simulator with black-box access to code.
- **Drawback:** Can only re-encrypt once.

End of Talk!

- Questions?

References

[Barak01] "On the (im)possibility of obfuscating programs", CRYPTO 2001

[Lynn04] "Positive results and techniques for obfuscation", EUROCRYPT 2004

[Wee05] "On obfuscating point functions", STOC 2005

[Varnovsky03] "On the possibility of provably secure obfuscating programs", PSI 2003, LNCS 2890.

[Goldwasser07] "On best possible obfuscation", TCC 2007

[Hohenberger07] "Securely obfuscating re-encryption", TCC 2007