S.P.E.E.D.

Signal Processing in the EncryptEd Domain

Introduction to Secure Multiparty Computation techniques

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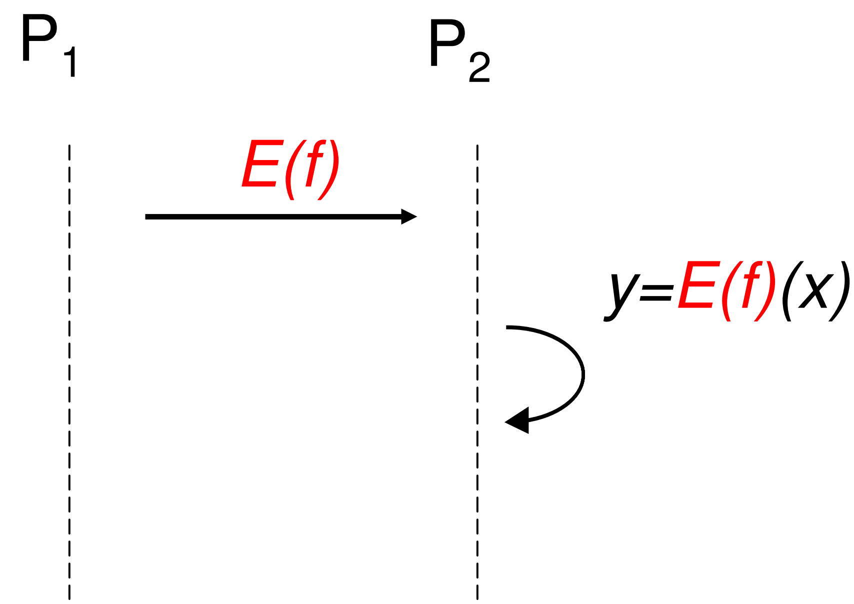
Outline

- Obfuscation
- Cryptocomputing
- Secure 2-party Computation
  - Yao's garbled circuit
- Secure n-party Computation
  - Secret sharing-based arithmetic circuit
- Practical feasibility

Different Scenarios – Obfuscation

- P_1 wants to protect his function
- P_1 gives to P_2 the "encrypted" function
- P_2 computes the function on any input

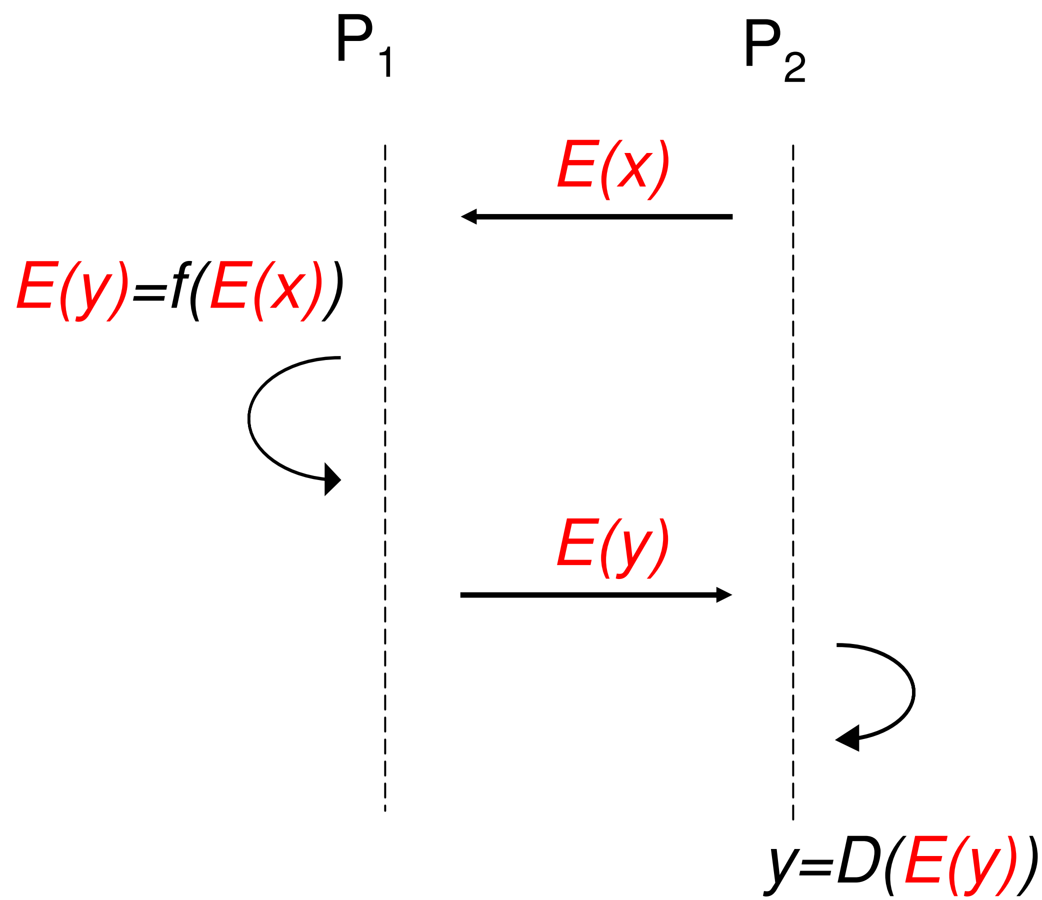


Obfuscation – state of the art

- What kind of obfuscation?
  - the attacker cannot learn more than from black-box access to the function
- General impossibility result
  - Barak et al. 2001
- Few positive results
  - Point functions, Re-encryption, …

Different Scenarios – Cryptocomputing

- P_2 gives to P_1 the encrypted input
- P_1 computes any function of it
- P_1 sends back the encrypted output
- P_1 decrypts his output



Homomorphic Encryption

- It's possible to compute on plaintexts just manipulating ciphertexts

$$E_{pk}(x) - E_{pk}(y) = E_{pk}(x \copyright y)$$

Multiplicative Homomorphic Encryption

$$E_{pk}(x)E_{pk}(y) = E_{pk}(xy)$$

- RSA

$$c_1 = x_1^e \mod n \qquad c_2 = x_2^e \mod n$$

$$c_1 c_2 = (x_1^e)(x_2^e) = (x_1 x_2)^e \mod n$$

Multiplicative Homomorphic Encryption

$$E_{pk}(x)E_{pk}(y) = E_{pk}(xy)$$

- ElGamal

$$c_1 = (g^{r_1}; x_1 h^{r_1}) \qquad c_2 = (g^{r_2}; x_2 h^{r_2})$$

$$c_1 c_2 = (g^{r_1 + r_2}; x_1 x_2 h^{r_1 + r_2})$$

Additive Homomorphic Encryption

$$E_{pk}(x)E_{pk}(y) = E_{pk}(x+y)$$

$$E_{pk}(x)^a = E_{pk}(ax)$$

- Modified ElGamal

$$c_1 = (g^{r_1}; g^{x_1}h^{r_1}) \quad c_2 = (g^{r_2}; g^{x_2}h^{r_2})$$

$$c_1 c_2 = (g^{r_1+r_2}; g^{x_1+x_2}h^{r_1+r_2})$$

Inefficient decryption!

Additive Homomorphic Encryption

$$E_{pk}(x)E_{pk}(y) = E_{pk}(x + y)$$

$$E_{pk}(x)^a = E_{pk}(ax)$$

- Paillier

$$c_1 = g^{x_1} r_1^n \mod n^2 \qquad c_2 = g^{x_2} r_2^n \mod n^2$$

$$c_1 c_2 = g^{x_1 + x_2} (r_1 r_2)^n \mod n^2$$

Cryptocomputing

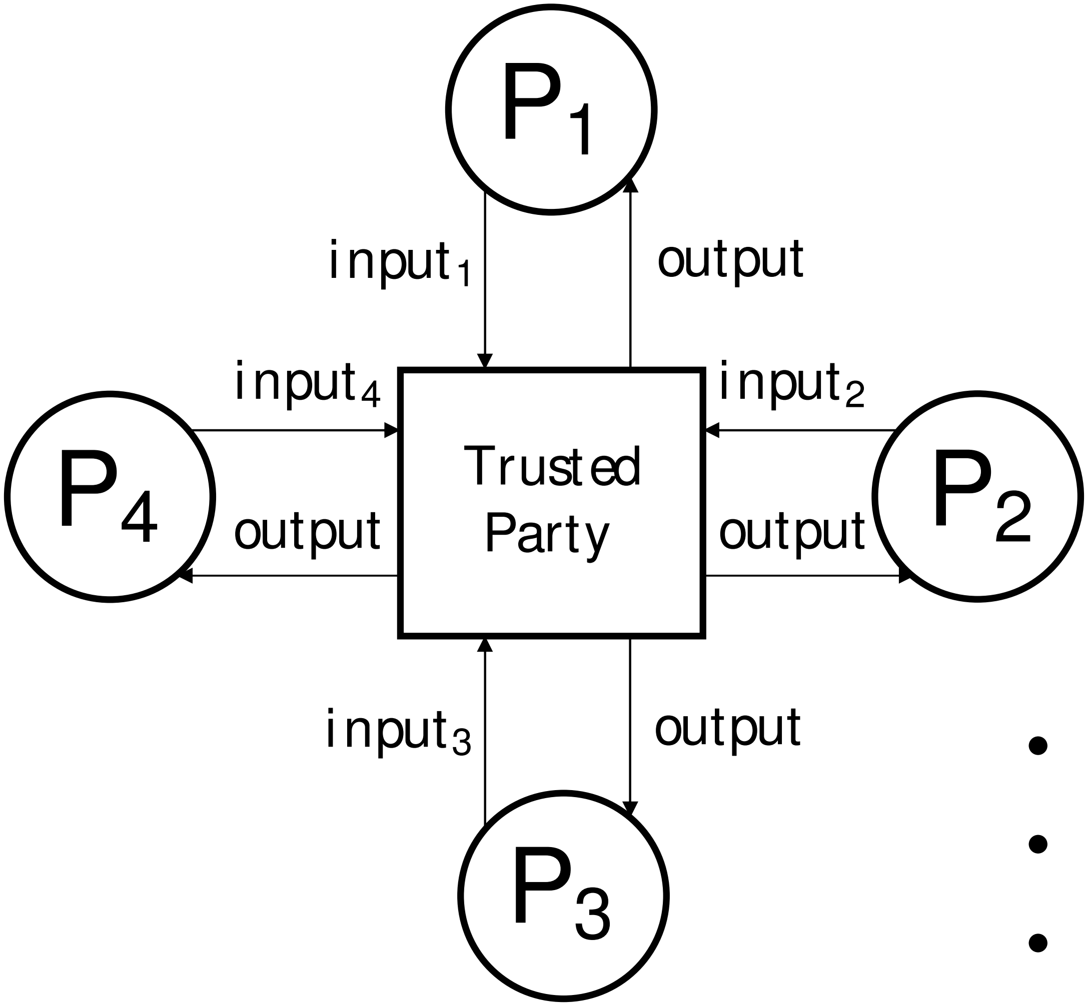
- Fully Homomorphic Cryptosystem?
- State of the art
  - Non-interactive Cryptocomputing for NC^1 Sander, Young 1999
  - the size of the ciphertext **doubles** after every operation
  - just for logarithmic-depth circuits

Interaction is needed?

- To compute any function in a secure way, you need to resort to Secure Multiparty Techniques

- Pros
  - General feasibility
  - Strong security guarantees
- Cons
  - Computational overhead
  - Communication overhead
  - All parties need to cooperate online

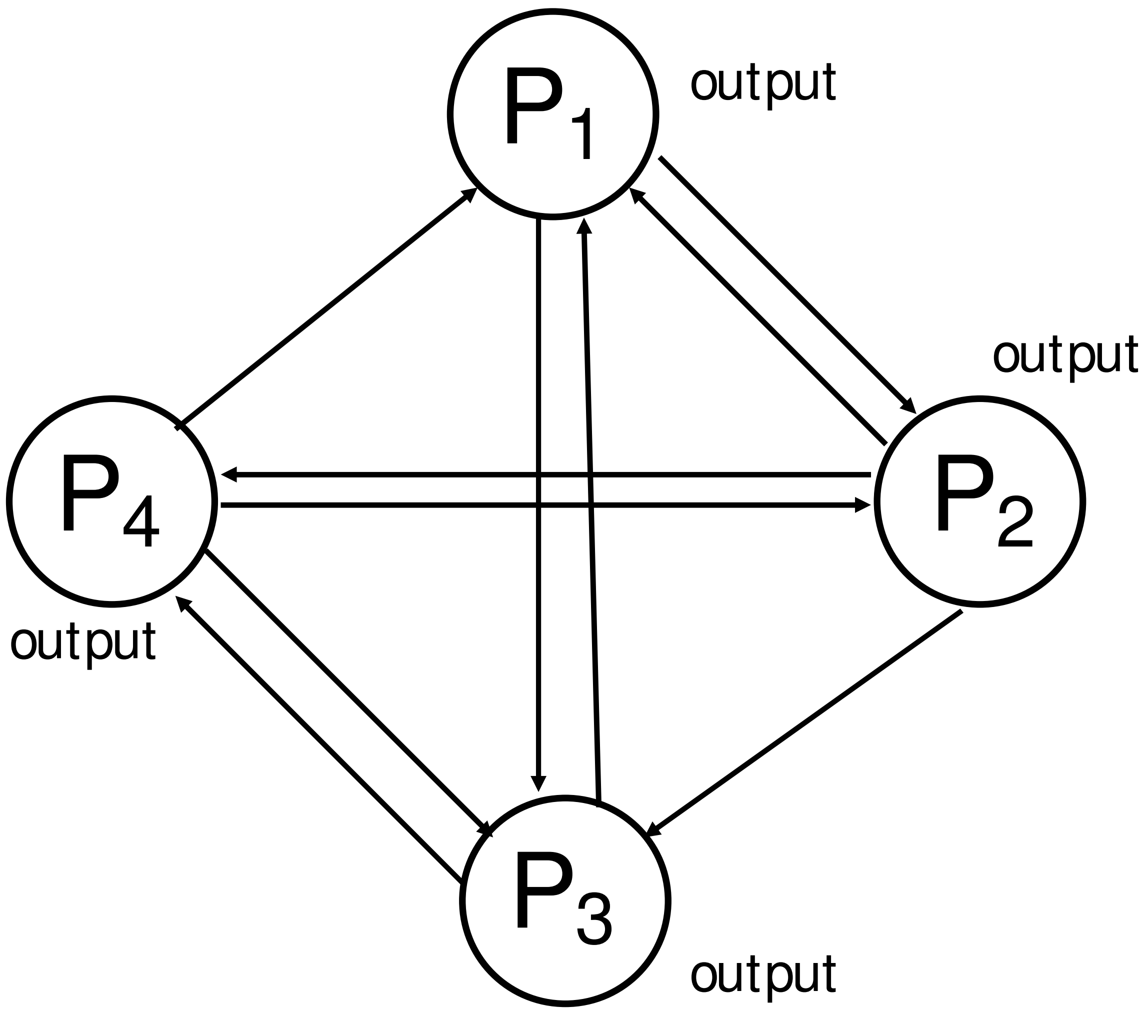
Secure Multiparty Computation



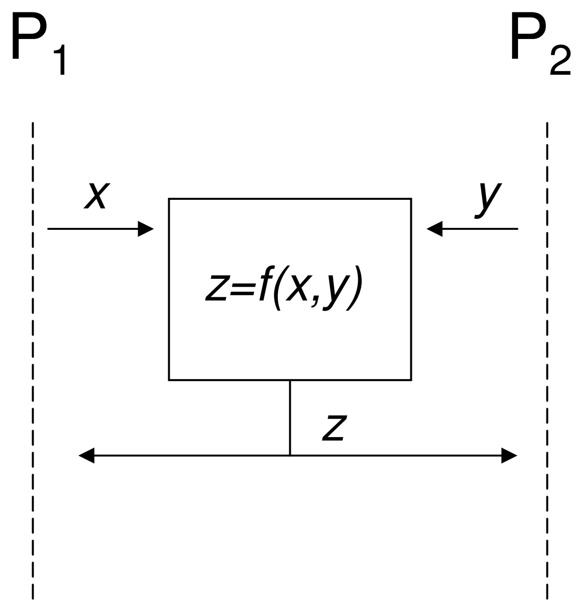
- Parties agree on a function to be computed
- They want to protect their inputs

- Auction
- Voting
- …

Secure Multiparty Computation



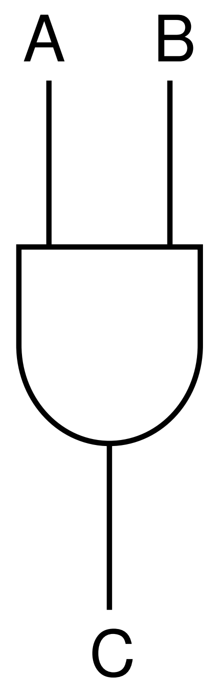
Secure 2-party Computation



- Yao's solution (1982):
  - P_1 "garbles" the circuit
  - P_2 evaluates the garbled circuit

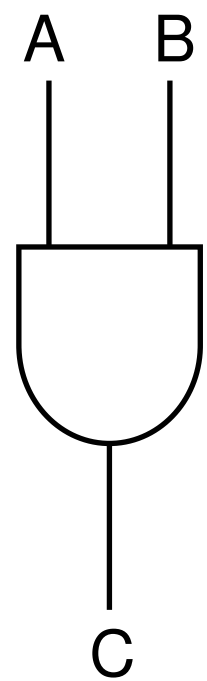
Yao's garbled circuits (1)

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1



Yao's garbled circuits (2)

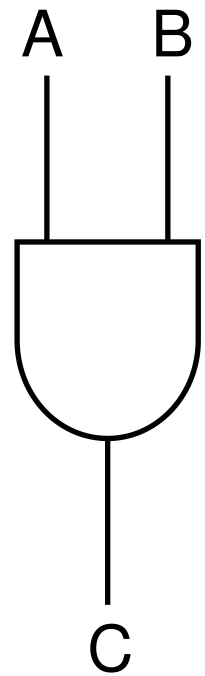
A	B	C
a_0	b_0	c_0
a_0	b_1	c_0
a_1	b_0	c_0
a_1	b_1	c_1



- P_1 selects a random string for every values, for all wires

Yao's garbled circuits (3)

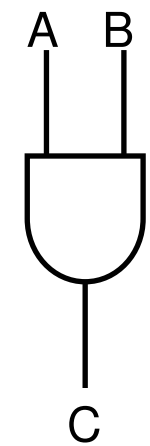
A	B	C
a_0	b_0	$E_{a0,b0}(c_0)$
a_0	b_1	$E_{a0,b1}(c_0)$
a_1	b_0	$E_{a1,b0}(c_0)$
a_1	b_1	$E_{a1,b1}(c_1)$



- P_1 encrypts the output using the inputs as a key
- P_1 permutes the table randomly

Yao’s garbled circuits (4)

C
$E_{a1,b0}(c_0)$
$E_{a1,b1}(c_1)$
$E_{a0,b0}(c_0)$
$E_{a0,b1}(c_0)$

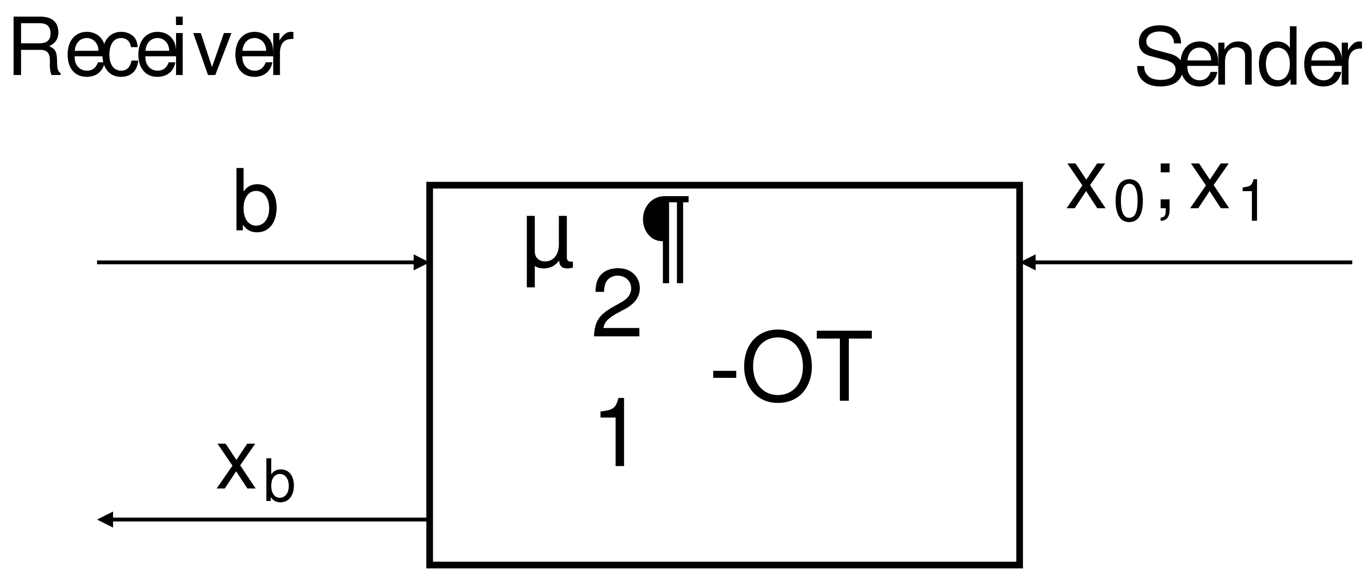


- P_1 sends to P_2 the garbled table
- P_1 sends the string corresponding to his input
  - It appears just as a random string to P_2
- P_2 needs the string associated to his input

Yao's garbled circuits (5)

- P_2 needs the string associated to his input
- P_2 doesn't want to reveal his input to P_1
- P_1 doesn't want to reveal both strings to P_2
  - Computing g(0,B) and g(1,B) P_2 will learn B
- Solution? Oblivious Transfer

1 out of 2 Oblivious Transfer



- Sender doesn't know which secret is chosen
- Receiver doesn't learn the other secret

A simple OT protocol

Receiver | **Sender**

$$c_0 = E(1-b)$$
$$c_1 = E(b)$$

(1,0) o (0,1)

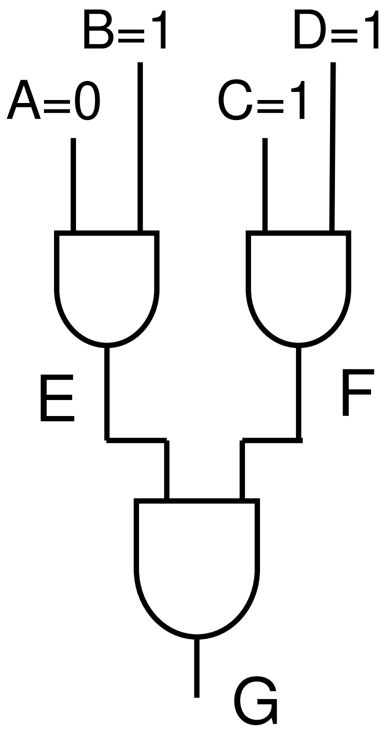
$$\xrightarrow{c_0;\, c_1}$$

$$x_b = D(d) \quad \xleftarrow{\quad d \quad} \quad d = c_0^{x_0} c_1^{x_1}$$

$$d = c_0^{x_0} c_1^{x_1} = E(1-b)^{x_0} E(b)^{x_1} = E((1-b)x_0 + bx_1)$$
$$= E(x_b)$$

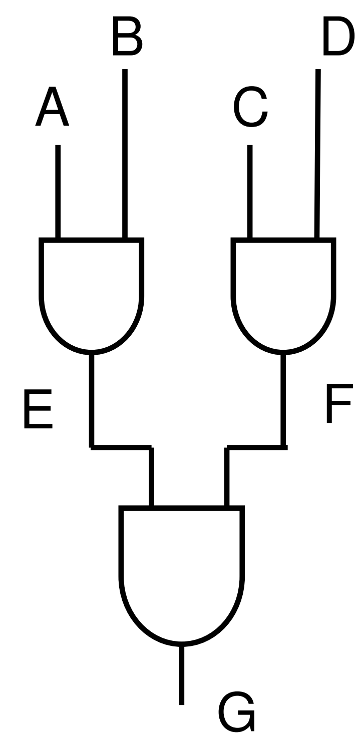
Yao's garbled circuits – Final protocol

- P_1 inputs: (A,C) = (0,1)
- P_2 inputs: (B,D) = (1,1)



Yao's garbled circuits – Setup

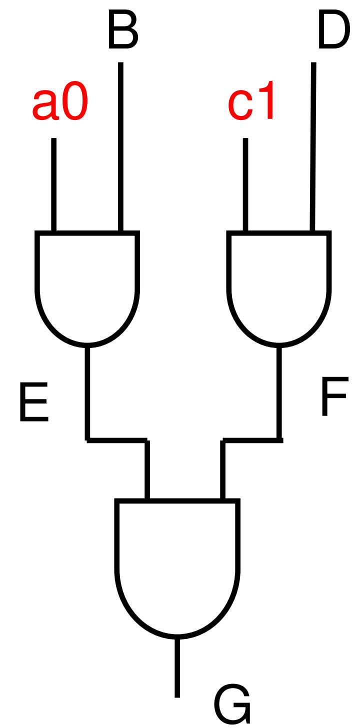
- P_1 prepares the garbled circuit
  - Assign a pair of secret strings to each wire
  - Encrypt the output of each gate with secret strings



- P_1 sends the garbled circuit to P_2

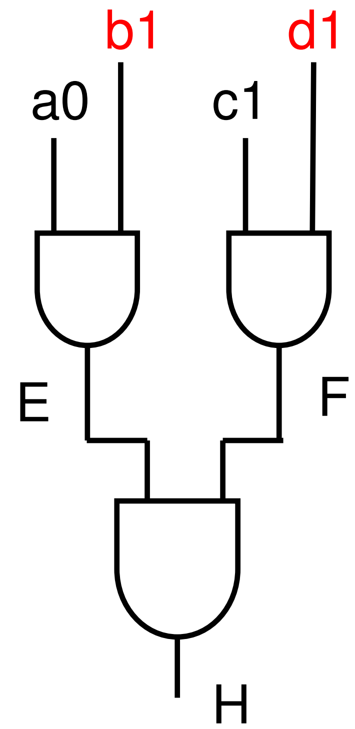
Yao's garbled circuits – Inputs exchange

- P_1 sends to P_2 the strings corresponding to his inputs,



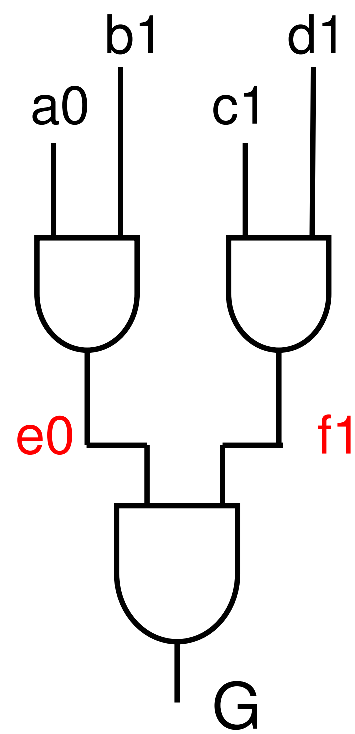
Yao's garbled circuits – Inputs exchange

- P_1 sends to P_2 the strings corresponding to his inputs,
- P_1-P_2 run Oblivious Transfer
  - P_2 obtains secret strings corresponding to his inputs



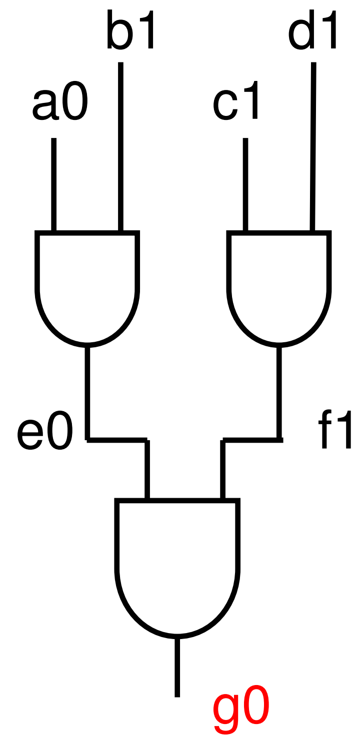
Yao’s garbled circuits – Evaluating

- P_2 uses the secret strings to decrypt the output of the first layer



Yao's garbled circuits – Evaluating

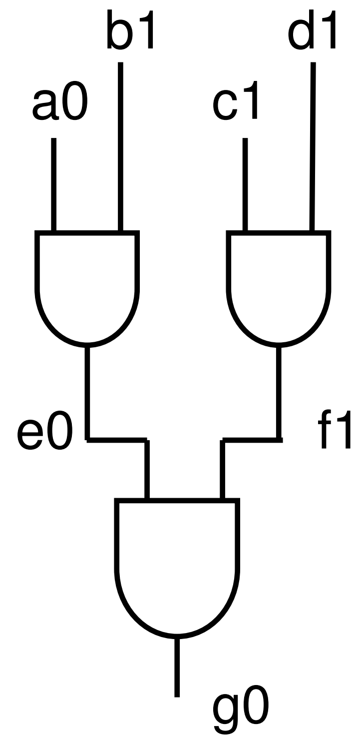
- P_2 uses the secret strings to decrypt the output of the first layer
- P_2 uses these strings to decrypt the second layer



Yao's garbled circuits – Decoding

- P_1 sends to P_2
  - <H(g0),0>
  - <H(g1),1>

  (H some hash function)



- P_2 evaluates f on the obtained string and learns the actual output

- P_2 communicates to P_1 the output

Yao's garbled circuits

- P_1 generates the garbled circuit
  - Assign random strings for each wire
  - Encrypt
  - Permute
- P_2 obtains random strings for his inputs with OT
  - Oblivious Transfer
- P_2 evaluate the circuit
  - Decoding layer by layer
- P_2 recover the outputs and sends it to P_1
  - Decoding table

Arithmetic circuits

- *Ben-Or, Goldwasser and Wigderson, 1988*
- *Chaum, Crépeau and Damgård, 1988*

- Idea
  - P_i has input x_i
  - P_i "shares" x_i between all parties → $[x_i]$
  - All parties jointly evaluate the circuit

    $$[y]=F([x_1],[x_2], \dots , [x_n])$$

  - They reconstruct $[y]$ → y

Secret sharing

- To share $x \in \{0, 1, \ldots, p\text{-}1\}$
  - Select a random t-degree polynomial g() such that f(0)=x
  - Sends f(i) to P_i
  - [x] = (f(1),f(2), … , f(n))
- Lagrange interpolation polynomial
  - t points: allow you to reconstruct the polynomial
  - t-1 points: don't give you any information about the polynomial
  - (There are *p* polynomials that passes for t-1 points)

Computing on secret sharing

- Addition (offline)
  - Compute [x+y] from [x] and [y]
  - f() such that f(0) = x
  - g() such that g(0) = y
  - (f+g)() such that (f+g)(0) = x+y

- Every party just add his shares

  → [x+y]=[x]+[y]

Computing on secret sharing

- Multiplication (online)
  - Compute [xy] from [x] and [y]
  - f() such that f(0) = x
  - g() such that g(0) = y
  - (fg)() such that (fg)(0) = xy
  - BUT: (fg) has degree 2t
- Interaction
  - is needed to compute h such that h(0)=xy and h has degree t

Arithmetic circuit

- From addition and multiplication you can compute any circuit
  - NOT: $1-a$
  - AND: ab
  - OR: $a + b - ab$
  - XOR: $1-(a-b)^2$

Practical feasability of general SMC

- Fairplay
  - implements the Yao's technique
  - *Malkhi et al. 2004*
- SIMAP
  - implements secret sharing based SMC with applications to food market
  - *national Danish Research Agency program*

Fairplay

```
program Millionaires {
  type int = Int<4>;  // 4-bit integer
  type AliceInput = int;
  type BobInput = int;
  type AliceOutput = Boolean;
  type BobOutput = Boolean;
  type Output = struct {
       AliceOutput alice, BobOutput bob};
  type Input = struct {
       AliceInput alice, BobInput bob};

  function Output out(Input inp) {
    out.alice = inp.alice > inp.bob;
    out.bob = inp.bob > inp.alice;
  }
}
```

Fairplay

- Execution time:
  - Bit-wise AND between 8 bit register: 2.14s
  - Comparison between 32 bit integers: 4.03s
  - Median of two sorted 10-elements arrays of 16 bits integers: 40.55s

SIMAP

- Secret sharing *efficient* primitives (not just addition and multiplication)
  - Damgård et al. 2005 – now
  - Comparison, equality, exponentiation, bit-decomposition etc.
- Language, compiler:
  - Nielsen and Schwartzbach 2007

SIMAP

```
declare client Millionaires:

  tunnel of sint netWorth;

  function void main(int[] args) {
   ask();
  }

  function void ask() {
   netWorth.put(readInt());
  }

  function void tell(bool b) {
   if (b) {
    display("You are the richest!");
   }
   else {
    display("Make more money!");
   }
  }
```

SIMAP

```
declare server Max:
 group of Millionaires mills;

 function void main(int[] args) {

  sint max = 0;
  sclient rich;

  for (client c in mills) {
   sint netWorth = c.netWorth.get();
   if (netWorth > max) {
    max = netWorth;
    rich = c;
   }
  }

  for (client c in mills) {
   c.tell(open(c==rich|rich));
  }
 }
```

Timing, comparison

	SIMAP		Fairplay
	(3,1)	(5,2)	Malkhi et al. (25)
PIR	113	327	8.65
$\cdot$	0.126	0.197	2.14
$<$	3.8	8.9	4.03

SIMAP – application

- December 2007
  - for the first time SMC techniques will be used in a real-world application
- Secure auction
  - find the price at which to trade a certain item while keeping the individual bids private
- Danish sugarbeet market
  - producers will use the system to find a fair market price at which to trade contracts for production of beets.

Thank you!
Questions?